Cambridge Assessment International Education

Notation List

For Cambridge International Mathematics Qualifications

For use from 2020

Mathematical notation

Examinations for CIE syllabuses may use relevant notation from the following list.

1 Set notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x : \ldots\}$	the set of all x such that …
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	the universal set
$\cup$	the universal set (for 0607 IGCSE International Mathematics)
A'	the complement of the set A
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p,\ q \in \mathbb{Z},\ q \neq 0\right\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{C}$	the set of complex numbers
(x, y)	the ordered pair x, y
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$(S, \circ)$	the group consisting of the set S with binary operation $\circ$

2 Miscellaneous symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\sim$	is distributed as
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leqslant$	is less than or equal to
$>$	is greater than
$\geqslant$	is greater than or equal to
∞	infinity
$\Rightarrow$	implies
$\Leftarrow$	is implied by
$\Leftrightarrow$	implies and is implied by (is equivalent to)

3 Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab	a multiplied by b
$a \div b$, $\frac{a}{b}$	a divided by b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\sqrt{a}$	the non-negative square root of a, for $a \in \mathbb{R}$, $a \geqslant 0$
$\sqrt[n]{a}$	the (real) nth root of a, for $a \in \mathbb{R}$, where $\sqrt[n]{a} \geqslant 0$ for $a \geqslant 0$
$\lvert a \rvert$	the modulus of a
$n!$	n factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}$ and $0 \leqslant r \leqslant n$

4 Functions

$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f} : A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the one-one function f
gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx, δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, …, nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\dot{x}$, $\ddot{x}$, …	the first, second, … derivatives of x with respect to t

5 Exponential and logarithmic functions

e	base of natural logarithms
e^x, $\exp(x)$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$, $\log_{10} x$	logarithm of x to base 10

6 Circular and hyperbolic functions

$\left.\begin{matrix}\sin,\ \cos,\ \tan \\ \operatorname{cosec},\ \sec,\ \cot\end{matrix}\right\}$	the circular functions
$\left.\begin{matrix}\sin^{-1},\ \cos^{-1},\ \tan^{-1} \\ \operatorname{cosec}^{-1},\ \sec^{-1},\ \cot^{-1}\end{matrix}\right\}$	the inverse circular functions
$\left.\begin{matrix}\sinh,\ \cosh,\ \tanh \\ \operatorname{cosech},\ \operatorname{sech},\ \coth\end{matrix}\right\}$	the hyperbolic functions
$\left.\begin{matrix}\sinh^{-1},\ \cosh^{-1},\ \tanh^{-1} \\ \operatorname{cosech}^{-1},\ \operatorname{sech}^{-1},\ \coth^{-1}\end{matrix}\right\}$	the inverse hyperbolic functions

7 Complex numbers

i	the imaginary unit, $i^2 = -1$
z	a complex number, $z = x + iy = r(\cos\theta + i\sin\theta)$
Re z	the real part of z, Re $z = x$
Im z	the imaginary part of z, Im $z = y$
$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$
arg z	the argument of z, arg $z = \theta$ where $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $x - iy$

8 Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the non-singular square matrix $\mathbf{M}$
det $\mathbf{M}$, $\lvert \mathbf{M} \rvert$	the determinant of the square matrix $\mathbf{M}$
$\mathbf{I}$	an identity (or unit) matrix

9 Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors in the directions of the Cartesian coordinate axes
$\begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	the vectors $x\mathbf{i} + y\mathbf{j}$ (in 2 dimensions) and $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ (in 3 dimensions)
$\lvert \mathbf{a} \rvert$, a	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$, AB	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10 Probability and statistics

$A, B, C, \ldots$	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A
$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
${}^{n}\mathrm{C}_{r}$	the number of combinations of r objects from n, ${}^{n}\mathrm{C}_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$
${}^{n}\mathrm{P}_{r}$	the number of permutations of r objects from n, ${}^{n}\mathrm{P}_{r} = \frac{n!}{(n-r)!}$
$X, Y, R, \ldots$	random variables
$x, y, r, \ldots$	values of the random variables $X, Y, R, \ldots$
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\mathrm{p}(x)$	probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
$\mathrm{f}(x)$	value of the probability density function of a continuous random variable X
$\mathrm{F}(x)$	value of the cumulative distribution function of a continuous random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}(\mathrm{g}(X))$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{G}_X(t)$	probability generating function for the discrete random variable X
$\mathrm{M}_X(t)$	moment generating function for the random variable X
$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
$\mathrm{Geo}(\mathrm{p})$	geometric distribution with parameter p
$\mathrm{Po}(\lambda)$	Poisson distribution with parameter λ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
ϕ	probability density function of the standardised normal variable $Z \sim \mathrm{N}(0, 1)$
Φ	cumulative distribution function of the standardised normal variable $Z \sim \mathrm{N}(0, 1)$
$\mathrm{H}_0, \mathrm{H}_1$	null and alternative hypotheses for a hypothesis test